AP Review Worksheet #5
2004-2005 AB Free Response

Name: Key

a) $2x + 8y\frac{dy}{dx} = 0 + 3x\frac{dy}{dx} + 3y$

$\frac{\frac{dy}{dx}(8y-3x)}{8y-3x} = \frac{3y-2x}{8y-3x}$

4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y-2x}{8y-3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

Check $(3,2)$ in original

$9 + 16 \stackrel{?}{=} 7 + 18$ ✓

$P(3,2)$

$\frac{dy}{dx}$ @ $P = 0$

$3y - 2x = 0$

$3y = 2(3)$

$y = 2$

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.

$\frac{d^2y}{dx^2} = \left[(8y-3x)(3\frac{dy}{dx} - 2) - (3y-2x)(8\frac{dy}{dx} - 3)\right]/(8y-3x)$

@ $(3,2) \to \left[(16-9)(-2) - (6-4)(-3)\right]/(16-9) \longrightarrow (-14+6)/7 \to \boxed{-\frac{8}{7}}$

concave down so max @ $P(3,2)$

5. Consider the curve given by $y^2 = 2 + xy$.

$2y\frac{dy}{dx} = 0 + x\frac{dy}{dx} + (1)y$

(a) Show that $\frac{dy}{dx} = \frac{y}{2y-x}$.

$\frac{\frac{dy}{dx}(2y-x)}{(2y-x)} = \frac{y}{2y-x}$

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

$\frac{1}{2} = \frac{y}{2y-x}$

$2y - x = 2y$

$x = 0,\ y = \pm\sqrt{2}$

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

$\frac{dy}{dx} = 0$

$\frac{y}{2y-x} = 0$

$y = 0$, but $y \neq 0$ in original

$0^2 \neq 2 + x(0)$

$0 \neq 2$

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

First find x when $y = 3$

$9 = 2 + 3x$

$\frac{7}{3} = x$

$2y\frac{dy}{dt} = 0 + x\frac{dy}{dt} + \frac{dx}{dt}y$

$2(3)(6) = \frac{7}{3}(6) + \frac{dx}{dt}(3)$

$\frac{36-14}{3} = \frac{dx}{dt}$

$\boxed{\frac{22}{3} = \frac{dx}{dt}}$

5. Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y-x}$.

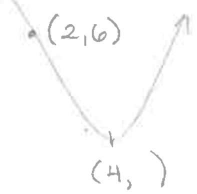
(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

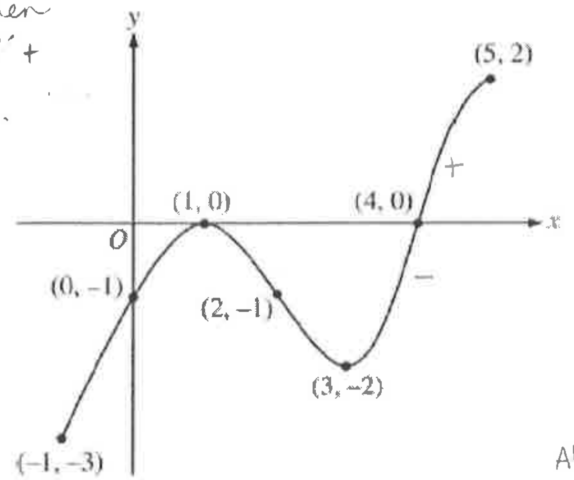
(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

repeat

b)
slopes are − on [−1, 4]
so f is decreasing, then
incr on [4, 5] where f′ +
Graph of f





Graph of f'

a) f' incr on $(-1, 0)$ so
concave up until $x = 1$
f' decr. on $(1, 3)$
PI @ $(1, 0)$
f' decr. $(1, 3)$ then
incr $(3, 5)$ so
PI @ $x = 3$

b) Extrema @ endpts of f or
where $f' = 0$
possibles @ $x = -1, 1, 4, 5$
Ab. Min where f' goes from − to +
at $\boxed{x = 4}$, Endpts are both
higher.
Ab. Max at
an endpt
b/c f'
never goes
from + to −
on $(-1, 5)$

4. The figure above shows the graph of f', the derivative of the function f, on the closed interval $-1 \le x \le 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.

where f' goes from incr to decr. or decr. to incr.

(a) Find the x-coordinate of each of the points of inflection of the graph of f. Give a reason for your answer.

(b) At what value of x does f attain its absolute minimum value on the closed interval $-1 \le x \le 5$? At what value of x does f attain its absolute maximum value on the closed interval $-1 \le x \le 5$? Show the analysis that leads to your answers.

(c) Let g be the function defined by $g(x) = xf(x)$. Find an equation for the line tangent to the graph of g at $x = 2$. (2, 6) slope → $g'(x) = x f'(x) + (1) f(x)$

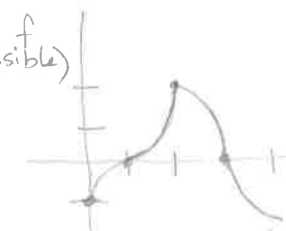
$g'(2) = 2 f'(2) + f(2)$
$= 2(-1) + 6 = 4$

$\boxed{y - 6 = 4(x - 2)}$

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	−1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	−3	Negative
$f''(x)$	−2	Negative	0	Positive	DNE	Negative	0	Positive

conc down conc up conc dwn conc up

f (possible)



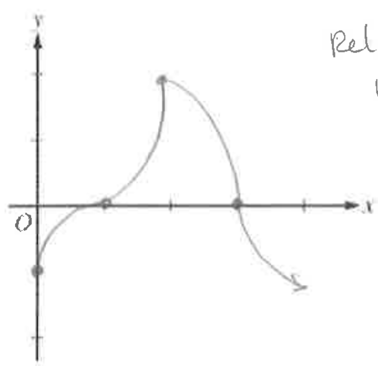
4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

No gap or hole so pointy @ $x = 2$

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

(Note: Use the axes provided in the pink test booklet.)



Extrema:
Relative Min. at $x = 0$ because f is increasing $[0, 2)$ since f' is +

Relative Max @ $x = 2$ where f changes from incr to decr because f' changes from + to −